# Antenna Selection for Spatial Multiplexing Systems with Linear Receivers

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***Abstract*—Future cellular systems will employ spatial multiplexing with multiple antennas at both transmitter and receiver to take advantage of large capacity gains. In such systems it will be desirable to select a subset of available transmit antennas for link initialization, maintenance or handoff. In this letter we present a criterion for selecting the optimal antenna subset when linear, coherent receivers are used over a slowly varying channel. We propose use of the post-processing SNR's (signal to noise ratios) of the multiplexed streams whereby the antenna subset that induces the largest minimum SNR is chosen. Simulations demonstrate that our selection algorithm also provides diversity advantage thus making linear receivers useful over fading channels.**

***Index Terms*—Diversity methods, MIMO systems, modulation, smart antennas.**

## I. Introduction

MULTIPLE-TRANSMIT multiple-receive antenna links are increasingly important because of their potential for extremely high spectral efficiencies [1]. One example of a spatial modulation technique for such MIMO (multiple-input multiple-output) systems is known as spatial multiplexing [2], [3]. It obtains high spectral efficiencies by dividing the incoming data into multiple substreams and transmitting each substream on a different antenna. The substreams are subsequently separated at the receiver by means of various techniques [2], [3].

Mobiles in future cellular systems supporting spatial multiplexing [4] will be capable of receiving substreams from multiple transmit antennas on one or more base stations. Simultaneous transmission from all available transmit antennas may be difficult due to hardware costs [5]. It is therefore of interest to select a subset of available antennas for transmission. Selection can occur upon *link initialization*, when the mobile determines from which antennas it wishes to receive substreams, for *link maintenance* when substreams are shifted to alternate antennas as the channel changes, and for *partial handoff* of some of the substreams between cells as the mobile moves.

In this letter we propose a selection criterion based on output SNR for selecting a subset of transmit antennas for spatial multiplexing systems that employ linear receivers. Although maximum likelihood (ML) receivers have superior performance [6], linear receivers offer a significant computational reduction. We compare the performance of our selection criterion with baseline ML and linear receivers over the flat-fading quasistatic channel in terms of symbol error rate. The results are surprising—with as little as one extra antenna—subset selection can dramatically improve the performance of linear receivers.

Selection has been considered in the past in the context of both transmit and receive diversity [7], [8]. Selection for multiple-transmit multiple-receive antenna systems was first presented in [5] based on an argument that it increases capacity. The selection criterion proposed therein is based on Shannon capacity and does not readily apply to spatial multiplexing with linear receivers. Use of output SNR to adapt rate in MIMO systems can be found in [9].

## II. Data and Channel Model

Consider a spatial multiplexing system with $M_t$ transmit antennas, $M_r$ receive antennas, and a $1 : M$ ($M_t > M$, $M_r \geq M$) multiplexer. The channel is flat-fading and slowly time varying. It is unknown at the transmitter but is known at the receiver. A low bandwidth, zero-delay, error-free feedback link provides limited channel information from the receiver to the transmitter.

The spatial multiplexer works as follows. At one symbol time, $M$ input symbols with unit energy, and from the same constellation, are multiplexed to produce the $(M \times 1)$ vector symbol $\mathbf{s}_n$ for transmission over $M$ transmit antennas. The subset of $M \leq M_t$ transmit antennas is determined by a selection algorithm operating at the receiver. Via the feedback path, the receiver sends the transmitter the optimal subset $p \in P$, where $P$ is the set of all possible $\binom{M_t}{M}$ subsets of transmit antennas.

Let $\mathbf{H}$ denote the $M_r \times M_t$ channel matrix and let $\mathbf{H}_p$ denote the $M_r \times M$ submatrix corresponding to transmit antenna subset $p$. The corresponding received signal is

$$\mathbf{x}_n = \sqrt{\frac{E_s}{M}}\,\mathbf{H}_p \mathbf{s}_n + \mathbf{v}_n \tag{1}$$

where $\mathbf{x}_n$ is $M_r \times 1$ and the noise vector $\mathbf{v}_n$ is $M_r \times 1$. The normalization is such that the maximum total power transmitted on $M$ antennas at one symbol time is $E_s$. The entries of $\mathbf{v}_n$ are i.i.d., $\mathbf{v}_n(k) \sim \mathcal{CN}(0, N_0)$, and independent over $k$. An $M \times M_r$ matrix equalizer $\mathbf{G}$ is applied to $\mathbf{x}_n$ to obtain an estimate of $\mathbf{s}_n$ as follows:

$$\hat{\mathbf{s}}_n = \mathbf{G}\mathbf{x}_n = \sqrt{\frac{E_s}{M}}\,\mathbf{G}\mathbf{H}_p \mathbf{s}_n + \mathbf{G}\mathbf{v}_n. \tag{2}$$

Manuscript received August 28, 2000. The associate editor coordinating the review of this letter and approving it for publication was Prof. R. Blum. The work of R. W. Heath was supported by Ericsson, Inc. through the Stanford Networking Research Center. The work of S. Sandhu was supported by a Ph.D. Fellowship from Hughes Research Laboratories, Malibu, CA.

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Publisher Item Identifier S 1089-7798(01)03657-2.

## III. Performance Analysis

First we derive expressions for the post-processing SNR of each of the $M$ multiplexed streams. Let $\mathbf{g}_k^*$ be the $k$th row of $\mathbf{G}$ and $\mathbf{h}_k$ be the $k$th column of $\mathbf{H}_p$. From (2) the post-processing SNR of the $k$th stream is

$$\mathrm{SNR}_k = \frac{E_s|\mathbf{g}_k^*\mathbf{h}_k|^2}{MN_0\|\mathbf{g}_k\|^2 + E_s\sum_{j\neq k}|\mathbf{g}_k^*\mathbf{h}_j|^2}. \quad (3)$$

For the MMSE (minimum mean-square error) receiver, $\mathbf{G} = [\mathbf{H}_p^*\mathbf{H}_p + N_0/E_s\mathbf{I}_M]^{-1}\mathbf{H}_p^*$ where $*$ denotes the Hermitian. After some computation (3) simplifies to

$$\mathrm{SNR}_k^{(\mathrm{MMSE})} = \frac{E_s}{MN_0[\mathbf{H}_p^*\mathbf{H}_p + N_0/E_s\mathbf{I}_M]_{kk}^{-1}} - 1.$$

For the ZF (zero-forcing) receiver $\mathbf{G} = \mathbf{H}_p^{\dagger}$, the pseudoinverse of $\mathbf{H}_p$, and (3) simplifies to

$$\mathrm{SNR}_k^{(\mathrm{ZF})} = \frac{E_s}{MN_0[\mathbf{H}_p^*\mathbf{H}_p]_{kk}^{-1}}. \quad (4)$$

An approximation to (4) can be obtained as follows. Let $\lambda(\mathbf{A})$ denote singular values of matrix $\mathbf{A}$. Then

$$\begin{aligned}\max_k[\mathbf{H}_p^*\mathbf{H}_p]_{kk}^{-1} &= \max_k \mathbf{e}_k^*[\mathbf{H}_p^*\mathbf{H}_p]^{-1}\mathbf{e}_k && (5)\\ &\le \max_{\|x\|^2=1}\mathbf{x}^*[\mathbf{H}_p^*\mathbf{H}_p]^{-1}\mathbf{x}\\ &= \lambda_{\max}([\mathbf{H}_p^*\mathbf{H}_p]^{-1})\\ &= \lambda_{\min}^{-2}(\mathbf{H}_p) && (6)\end{aligned}$$

where $\mathbf{e}_k$ is the $k$th column of $\mathbf{I}_M$ and (6) follows from the Rayleigh–Ritz theorem. This result can then be used to bound (4) as

$$\mathrm{SNR}_{\min}^{(\mathrm{ZF})} \ge \lambda_{\min}^2(\mathbf{H}_p)\frac{E_s}{MN_0} \quad (7)$$

which is a simple function of the channel. The expression in (7) confirms the intuition that the performance of linear receivers should improve as the smallest singular value of the channel increases.

Now performance in terms of symbol error rate can be computed as follows. Define $\mathrm{SNR}_{\min} := \min_k \mathrm{SNR}_k$ and $k_{\min} := \arg\min_k \mathrm{SNR}_k$. Depending on the input constellation, the exact probability of symbol error $P_k$ can be computed from published formulas [10] for a given $\mathrm{SNR}_k$. We write the probability of vector symbol error (probability that at least one substream symbol is in error) as

$$\begin{aligned}P &= 1 - \prod_{k=1}^{M}(1 - P_k) && (8)\\ &\le 1 - (1 - P_{k_{\min}})^M && (9)\\ &\approx MP_{k_{\min}} && (10)\\ &\le MN_eQ\left(\sqrt{\mathrm{SNR}_{\min}\frac{d_{\min}^2}{2}}\right) && (11)\end{aligned}$$

where $d_{\min}^2$ is the squared minimum distance and $N_e$ is the average number of nearest neighbors of the per-antenna constellation. In (8) we rewrite the probability of vector symbol error as one minus the probability of no errors and in (9) we upper-bound this quantity by $P_{k_{\min}} := \min_k P_k$. For low values of $P_{k_{\min}}$, we get the approximation in (10) and apply the NNUB (nearest neighbor union bound) to obtain (11).

At the receiver, the performance in (11) is a function only of $\mathrm{SNR}_{\min}$. Computation of $\mathrm{SNR}_{\min}$ requires a search over all equalizers $\mathbf{g}_k$ in (3) for all channel subsets $p \in P$ in (1). In (7) we provide a lower bound for $\mathrm{SNR}_{\min}$ that only requires searching over all subsets $p \in P$.

## IV. Antenna Selection Criteria

Performance of spatial multiplexing with linear receivers depends on the $\mathrm{SNR}_{\min}$ induced by the particular subset of transmit antennas as shown in (11). One antenna subset selection criterion (SC) is therefore obtained as follows.

*SC1—Maximum Post-Processing SNR:* For every subset of transmit antennas $p \in P$ compute $\mathbf{G}$ and the corresponding $\mathrm{SNR}_{\min}$. Choose the subset with the largest $\mathrm{SNR}_{\min}$.

Computation of the above requires first the equalizer $\mathbf{G}$, then the $\mathrm{SNR}_{\min}$. An alternative selection criterion, based solely on the channel, follows from (7).

*SC2—Maximum Minimum Singular Value:* For every subset of transmit antennas $p \in P$ compute $\lambda_{\min}$ corresponding to $\mathbf{H}_p$. Choose the subset with the largest $\lambda_{\min}$.

Finally, for reference we provide the capacity-based selection criterion proposed in [5].

*SC3—Maximum Capacity:* For every subset of transmit antennas $p \in P$ compute $C_p := \log\det(\mathbf{I}_M + E_s/MN_0\mathbf{H}_p^*\mathbf{H}_p)$. Choose the subset with the largest $C_p$.

Note that SC3 is based on a general capacity formula and is not specialized to linear receivers. We expect that for certain channels optimal selection in terms of capacity will yield suboptimal performance for the linear receivers under consideration.

## V. Simulations

In this section the three selection criteria are compared via Monte Carlo simulations. Following in the previous development, performance is measured in terms of VSER (vector symbol error rate) for a frame of 100 vector symbols from QAM constellations averaged over 10 000 frames. The coefficients of channel $\mathbf{H}$ are i.i.d. (independent, identically distributed) circular complex Gaussian random variables with variance 0.5 in each dimension, i.e., $h_{i,j} \sim \mathcal{CN}(0, 1)$. Channel realizations are i.i.d. from frame to frame. In the following, let $X \times Y$ denote a spatial multiplexing system with $X$ transmit antennas and $Y$ receive antennas. We consider the following pairs: 3×2, $4 \times 2$, and 4×3. For an $X \times Y$ system, $Y$ antennas are selected at the transmitter using each of the three selection criteria (SC1 to SC3). Due to space constraints we consider only the ZF equalizer. Similar performance improvements have been observed with the MMSE receiver. For reference we compare with three baseline techniques (1×1, 2×2 ML, and 2×2 ZF).

In Fig. 1 we plot the VSER for a 3×2 system at a spectral efficiency of 4 bits/s/Hz. Without additional transmit antennas to choose from, the 2×2 ZF system does not obtain any diversity advantage over the 1×1 system. With optimal selection, however, the 3×2 linear system obtains a diversity advantage

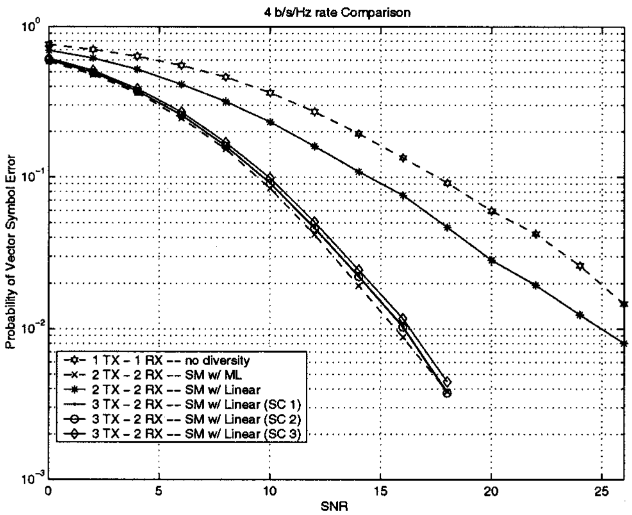


Fig. 1. 3×2 system.

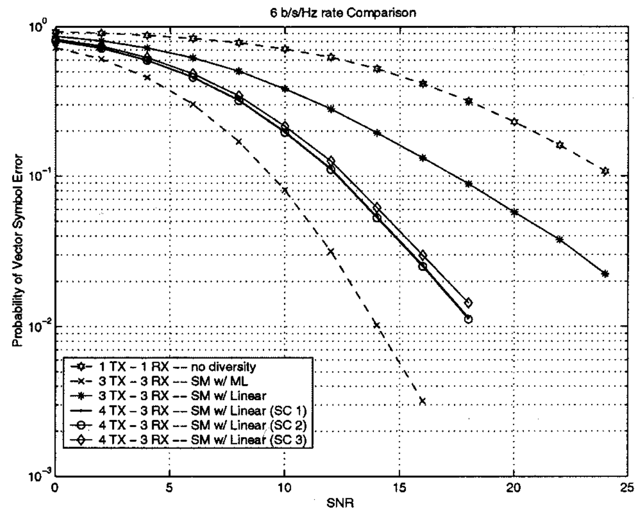


Fig. 3. 4×3 system.

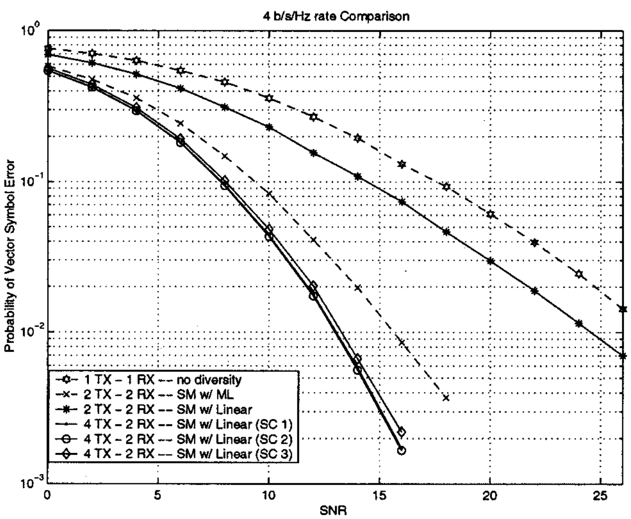


Fig. 2. 4×2 system.

rivaling that of the 2×2 ML receiver. At a VSER of $10^{-2}$, the 3×2 system is 0.5 dB short of the 22×2 ML and a significant 9 dB better than the 2×2 linear receiver. Among the three selection criteria SC1 gives the best performance while SC2 is about 0.5 dB worse. As expected SC1 has better performance (although slight) than SC3 verifying the earlier claim that capacity-based selection is not optimal in terms of VSER for the linear receiver.

In Fig. 2 we repeat the above experiment for a 4×2 system. Note that the extra two transmit antennas give an effective third order diversity advantage outperforming the 2×2 ML receiver by 3 dB at a VSER of $10^{-2}$.

In Fig. 3 we plot the performance of antenna selection in a 4×3 system with spectral efficiency $R = 6$ bits/s/Hz. Observe for the 3×3 system that as predicted by [7], the ML receiver gives a diversity advantage of $M_r = 3$ while the linear receiver exhibits no diversity.

Selection with an extra transmit antenna appears to give a second-order diversity advantage. We conjecture that with optimal selection, the diversity advantage for a zero-forcing receiver with $M = M_r$ streams and $M_t$ transmit antennas is $M_t - M + 1$. Proof of this conjecture is currently under investigation.

## VI. Conclusions

Future multiple-antenna cellular systems will employ spatial multiplexing to take advantage of large capacity gains. In such systems it will be necessary to select from a plurality of transmit antennas during link initialization, maintenance, and handoff. In this letter we derived an SNR-based criterion for antenna selection when a linear receiver is used. We showed via simulations that antenna selection is also an inexpensive way to obtain diversity over a multiple antenna fading channel. Availability of additional transmit antennas for selection can therefore improve the performance of linear receivers in MIMO systems.

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